Week 6: Extra Problems

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About These Problems

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6.1 Linear Regression

We are given that $X \sim N(0,1)$ and that Y is linear function of X given by $Y = \rho X + \varepsilon$, where $|\rho| < 1$ and $\varepsilon \sim N(0, 1 - \rho^2)$. We are also given that $\varepsilon \perp X$.

Problem 1: Calculate $E[Y|X = x]$.

Problem 2: Calculate $\text{Var}[Y|X = x]$.

Problem 3: Calculate the joint density $f(x, y)$ based on the chain rule $f(x, y) = f(x)f(y|x)$.

Problem 4: Calculate $E[Y]$

Problem 5: Calculate $\text{Var}[Y]$

Problem 6: Calculate $\text{Cov}(X, Y)$

6.2 Transformation of Random Variables

Let $U \sim Unif[0, 1]$ and let $X = -\log U$.

Problem 1: Calculate the cumulative density function $F(x) = P(X \leq x)$.

Problem 2: Calculate the probability density function $f(x) = F'(x)$.