Stats100A **Summer 2024**

Week 5: Extra Problems

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About These Problems

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5.1 Expectation & Variance

Problem 1: I flip a fair coin 10 times. What is the expected number of heads?

Problem 2: How many times would I expect to flip a fair coin until I see a heads?

Problem 3: How many times would I expect to roll a die until I see a 5?

Problem 4: X is a discrete random variable with distribution $q(x)$ and assumes values from a up to $a+n$. What is it's expectation?

Problem 5: X is a continuous random variable with distribution $q(x)$, with $q(x) > 0$ for $x \in [a, b]$, otherwise it's 0. What is the expectation of X ?

Problem 6: Given a random variable X, what is it's variance? Can you express the variance in two different ways?

Problem 7: Is it the case that $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$?

Problem 8: Does $E(XY) = E(X)E(Y)$?

Problem 9: I roll a fair 6-sided die once. Whatever value it lands on, call it a. Now roll a dice and take the sum of the face values rolled and call it b. What is $E(b)$?

Problem 10: Let X be a nonnegative integer-valued random variable and k a nonnegative constant. Show that $P(X \geq k) \leq \frac{E(X)}{k}$.

Problem 11: Let X be a nonnegative random variable and k a nonnegative constant. Show that $P(|X - E(X)| \geq k) \leq \frac{\text{Var}(X)}{k^2}$

Problem 12: Let X an nonnegative random variable that only takes on integer values. Show that $P(X > 0) \leq E(X)$

Problem 13: Let X an nonnegative random variable (not always 0) that only takes on integer values. Show that $P(X > 0) \geq \frac{(E(X))^2}{E(X^2)}$