Stats100A **Summer 2024**

Week 3: Extra Problems

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About These Problems

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3.1 Basic Examples

Problem 1 (Determining Coin Bias): I give you a coin and I want you to determine if it is biased or not. What can you do to try and determine the coins bias?

Problem 2 (Darts In One-Half): I have a unit square $[0,1]^2$ and I toss 10 darts in the unit square. How many darts would you expect to land in the left-half of the square?

Problem 3 (Measuring a Cancer Cell): I have a $2D$ slice of a cell on a plane and I want to measure a cross-sectional area of interest, let's call it $|A|$. The issue is that $|A|$ is highly irregular and my microscopes are too weak to view it. However, I can shoot X-rays at the cell and the area of interest give a signal response that I can measure. What can I do with this setup to measure $|A|$, given I know that the cells entire cross-sectional area is $|\Omega|$?

3.2 Tricky To Deduce Probabilistic Phenomena

Problem 4 (HH v.s. HT): I have a fair coin and I keep flipping until I see HH or HT. Should it take more flips on average until I see HH, or more flips for HT, or should it be the same?

Problem 5 (Jane Street's Some Off Square): A circle is randomly generated by sampling two points uniformly and independently from the interior of a square and using these points to determine its diameter. Approximate the probability that the circle has a part of it that is off the square?

3.3 Estimators

Problem 6 (Buffon's Needle): We have parallel lines that are separated 1-unit away from each other and they span the entire plane.

Problem 7 (Laplace's Needle): We have a set of parallel lines that are separated 1-unit away from each other. Additionally, we have another set of parallel lines, orthogonal to the first set of parallel lines, that are also spaced 1-unit apart. So the plane is effectively covered by a grid of unit squares. What is the probability that when tossing a needle that it crosses a line?