

## Week 4: Challenge Problems 4

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## About These Problems

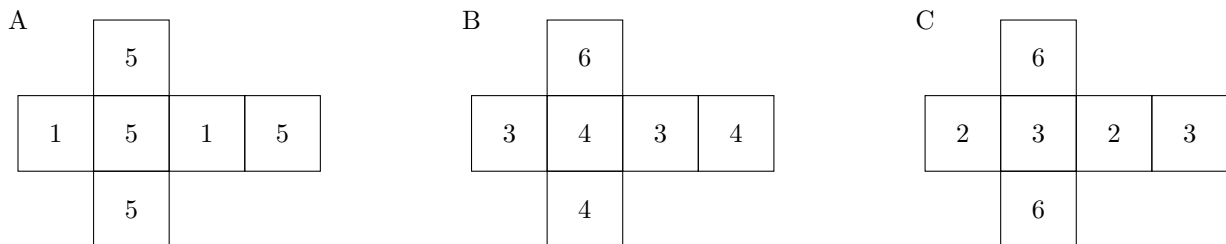
- This will not be graded. And it's not mandatory or necessary to do these problems.
- You should focus on homework and lectures to do well in this class.
- These are merely meant to be supplementary challenge problems for those who want them.
- Consult Andrew Lizzarraga: andrewlizzarraga at g.ucla.edu for question or solutions.

## 4.1 Welcome back challenger! I wait for you conditionally so.

**Problem 1 (Repulsive Probability):** The event  $A$  is said to be repelled by the event  $B$  if  $P(A|B) < P(A)$ , and it is said to be attracted to  $B$  if  $P(A|B) > P(A)$ . Show that if  $B$  attracts  $A$ , then  $A$  attracts  $B$ , and  $B^c$  repels  $A$ . If  $A$  attracts  $B$ , and  $B$  attracts  $C$ , does  $A$  attract  $C$ ?

**Problem 2 (Around The Quarter):** A quarter is glued to a tabletop. You then place another quarter tangent to the tabletop quarter and begin rotating it around the tabletop quarter. You keep rotating it until your quarter reaches the point it started at. How many times will your quarter rotate?

**Problem 3 (Rigged Dice):** I have dice,  $A, B, C$  with face values depicted below:



Show that  $P(A > B) = P(B > C)$ . Is it the case that  $P(A > C) = P(A > B) = P(B > C)$ ?

**Problem 4 (Coin Game I):** Player  $A$  has 100 fair coins and player  $B$  has 101 fair coins. Both of them toss their respective coins simultaneously and count the number of heads they each respectively received. Whoever has more heads is declared the winner. What's the probability that player  $B$  beats player  $A$ .

**Problem 5 (Coin Game II):** Players  $A$  and  $B$  are playing a game where they take turns flipping a biased coin, with probability  $p$  of landing heads (and winning). Player  $A$  starts the game, and then the players pass the coin back and forth until one person flips heads and wins.

What is the probability that  $A$  wins?