

$\pi$ -Day Puzzles*Author: Andrew Lizzarraga***Problem 1: Buffon's Needle Experiment**

A table is lined with parallel strips of width  $t$  and we drop a needle of length  $l < t$  onto the table. What is the probability the needle crosses a line?

**Problem 2: A Bent Needle**

Suppose now that the needle from problem 1 is bent into some planar curve shape. For example an "S" shape or a "C" shape. What is the probability that the bent needle crosses a line?

**Problem 3: Two grids of lines**

Now the table has a grid of parallel lines spaced  $a$  apart and another set of parallel lines (perpendicular to the first set) are imposed on the table and are spaced  $b$  apart. We drop a needle with length  $l < \min(a, b)$  on the table. What is the probability the needle intersects a line?

**Problem 4: Crossing The Line**

We have a table with parallel lines spaced 1 unit apart. We form a cross by welding two unit needles at their midpoints and drop it on the table. Let  $X$  denote the number of intersections the cross makes with a parallel line. What is  $\mathbb{E}[\frac{X}{2}]$ ?

**Problem 5: X-Ray Machine**

An individual inadvertently swallowed a needle and is examined via an X-Ray machine. If the needle exhibits no preference for direction in the gut, what is the distribution of the length of its image on the X-Ray plate?

**Problem 6: Digits of Pi**

Some people believe that the successive digits in the decimal expansion of  $\pi = 3.14159\dots$  are "uniformly" distributed. By this, they mean that the appearance of these digits are equally likely to be of any digit from 0 to 9. However, someone notices that starting with the 24,658,601<sup>st</sup> digit, there is a run of nine successive 7's. Is this information consistent with the hypothesis of a uniform distribution?